Magmons – magmatic solitons

The problems below use MATLAB code provided. To run the code, start MATLAB and navigate to the directory where the code is stored.

The goal of these exercises is to familiarise you with the concept of compaction stresses in magma dynamics. Recall that the compaction pressure is defined as

$$\mathcal{P} = \xi \boldsymbol{\nabla} \cdot \mathbf{v}_m,$$

and that for simplicity in developing the compaction problems, we have taken $\xi = 1$. So the compaction pressure is the pressure that drives divergence of the solid matrix (therefore it should really be called the *decompaction* pressure, but nevermind).

1. At the MATLAB prompt type `PropagateSolitaryWaves` and press enter. A plot window should appear with a plot of a solitary wave. This is the initial condition for a time-dependent solution of the equations

$$\phi_t = \mathcal{C}, \tag{1a}$$
$$[\phi^n \mathcal{C}_z]_z - \mathcal{C} = [\phi^n]_z, \tag{1b}$$

   where $\mathcal{C} = \phi_0^{-1} W_z$ is the *compaction rate* (actually it should be called the *decompaction* rate). By pressing any key, the evolution of this initial condition is computed and an animation appears in the plot window, showing the propagation of the wave.

   Note the axis labels, the domain size, and the range of the solution. Explain the style and direction of propagation. What causes the wave to propagate? Which way is magma flowing and how fast?

2. Now we compare the solitary wave with displacement waves on a stretched string. At the MATLAB prompt type `StretchedStringWaves` and press enter. Again you see the initial condition. Note that it has the same shape as the solitary wave from the first question. What is the difference between this initial condition and that of problem 1? What is the evolution equation for the displacement of a stretched string? Make a comparison between this equation and the set of equations (1).

   Press any key to watch the string displacement evolve in time.

3. How is the evolution of your initial condition different for the displacement of a stretched string and for the porosity of the partially molten mantle? What are the differences in the physics that explain this? What are the boundary conditions in each case? How does this manifest itself in the behaviour of the solutions?

4. The MATLAB code file `SolitaryWaveProfile.m` is used to compute the analytical profile for a solitary wave. Examine the contents of this file and identify, in the lecture notes, the equation(s) that are used. Experiment with running the code until you have produced a solitary wave of amplitude 3 that is amply spaced from the edges of the domain. Why don't you have the option to select the wavelength of the solitary wave?

5. Use `PropagateSolitaryWaves` to investigate how two initially distinct solitary waves of *different amplitude* interact. You can use `SolitaryWaveProfile` to generate the waves, but be sure that you combine them such that the background porosity is still unity. Comment on what you see. What is happening to the magma in the larger wave as it interacts with the smaller wave?

6. Use `PropagateSolitaryWaves` to investigate what happens when you start your numerical model with a Gaussian porosity perturbation, instead of a solitary wave solution. Recall that a Gaussian has the form

$$g(x) = A \exp\left[-\frac{(x - x_0)^2}{2\sigma^2}\right],$$

where A is the amplitude, x_0 is the centre, and σ is the standard deviation. How does the behaviour depend on A and σ?

7. Examine and run the MATLAB script `SolitaryWaveStep.m`. Describe the starting condition and its evolution. Describe the flow of magma, and how it differs from the wave propagation. Explain why this interesting behaviour occurs. Adjust the amplitude of the initial condition and describe the difference in behaviour.